

## Capabilities

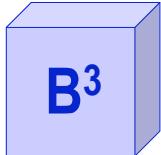
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- Motivating model: multielement array, noiseless environments
- **Extension to noisy environments**
- Extension to other linear processor structures

## Extension to Noisy Environments

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- Noise and background interference primary limit to performance of any interference excision system (after fundamental limit due to number of removable interferers)
  - By definition, nonremovable
  - Can only be suppressed by increasing gain relative to SOI (beamsteering gain)
  - Limits ultimate excision performance against strong interferers as well
- Motivates methods that can optimally trade interference excision and noise suppression of the system
- Primary excision metric for linear systems is signal-to-interference-and-noise ratio (SINR)
  - Provides common-sense measurement of quality of interference excision solution
  - Not the only (or even the best) metric, however, is tractable and easy to analyze for realistic systems
  - Alternate metrics include mean-square error (MSE), bit-error-rate (BER) and symbol-error-rate (SER)
  - Equivalent parameter estimation metrics include measurement error bias and variance
- Possesses simple and tractable upper bound — maximum-attainable SINR
  - Equivalent alternate bounds include minimum MSE (MMSE — especially useful in multipath environments) and Cramer-Rao bound (CRB — especially useful for parameter estimates)
  - Bounds based on information theory (e.g., channel capacity) gaining in importance, especially for cooperative system and nonlinear excision techniques
- Important to differentiate quality metrics from optimization methods — a metric may not be directly observable (although it may be estimable), and may not yield a usable optimization method. However, it can at a minimum provide a useful means for understanding the excision capability of a given system



## Signal Reception Model, Noisy Environment

$$\mathbf{x}_{\text{ADC}}(n) = \mathbf{h}_{\text{SOI}} s_{\text{SOI}}(n) + \mathbf{H}_{\text{SNOI}} \mathbf{s}_{\text{SNOI}}(n) + \boldsymbol{\varepsilon}_{\text{ADC}}(n), \quad \left\{ \begin{array}{l} \boldsymbol{\varepsilon}_{\text{ADC}}(n) = M_{\text{feed}} \times 1 \text{ sample } n \text{ receive noise at ADC output} \\ \qquad \qquad \qquad = \boldsymbol{\varepsilon}_{\text{sys}}(n) + \boldsymbol{\varepsilon}_{\text{bck}}(n) \\ \boldsymbol{\varepsilon}_{\text{sys}}(n) = M_{\text{feed}} \times 1 \text{ system noise,} \\ \qquad \qquad \qquad \text{generated inside the receiver} \\ \boldsymbol{\varepsilon}_{\text{bck}}(n) = M_{\text{feed}} \times 1 \text{ background noise and interference,} \\ \qquad \qquad \qquad \text{generated outside the receiver} \end{array} \right.$$
$$= \mathbf{h}_{\text{SOI}} s_{\text{SOI}}(n) + \mathbf{i}_{\text{ADC}}(n), \quad \mathbf{i}_{\text{ADC}}(n) = M_{\text{feed}} \times 1 \text{ interference at ADC output}$$

$$\mathbf{w}_{\text{SOI}}^H \mathbf{x}_{\text{ADC}}(n) = (\mathbf{w}_{\text{SOI}}^H \mathbf{h}_{\text{SOI}}) s_{\text{SOI}}(n) + (\mathbf{w}_{\text{SOI}}^H \mathbf{i}_{\text{ADC}}(n))$$

$$= h_{\text{SOI}} s_{\text{SOI}}(n) + \boldsymbol{\varepsilon}_{\text{SOI}}(n), \quad \left\{ \begin{array}{l} h_{\text{SOI}} = \text{Channel gain through linear combiner} \\ \qquad \qquad \qquad = \mathbf{w}_{\text{SOI}}^H \mathbf{h}_{\text{SOI}} \\ \boldsymbol{\varepsilon}_{\text{SOI}}(n) = \text{Residual interference at linear combiner output} \\ \qquad \qquad \qquad = \mathbf{w}_{\text{SOI}}^H \mathbf{i}_{\text{ADC}}(n) \end{array} \right.$$

# Signal-to-Interference-and-Noise Ratio (SINR)

$$\gamma_{\text{SOI}} \triangleq \frac{|h_{\text{SOI}}|^2 R_{s_{\text{SOI}} s_{\text{SOI}}}}{R_{\varepsilon_{\text{SOI}} \varepsilon_{\text{SOI}}}}$$

$$= \frac{\mathbf{w}_{\text{SOI}}^H \mathbf{h}_{\text{SOI}}}{\mathbf{w}_{\text{SOI}}^H \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}} \mathbf{w}_{\text{SOI}}} |^2 R_{s_{\text{SOI}} s_{\text{SOI}}}$$

Average over time index  $n$ ,  
 $\langle \bullet \rangle \triangleq$  or statistical expectation  
 (ergodic processes)

$$R_{s_{\text{SOI}} s_{\text{SOI}}} \triangleq \left\langle |s_{\text{SOI}}(n)|^2 \right\rangle$$

$$R_{\varepsilon_{\text{SOI}} \varepsilon_{\text{SOI}}} \triangleq \left\langle |i_{\text{SOI}}(n)|^2 \right\rangle$$

$$\mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}} \triangleq \left\langle |\mathbf{i}_{\text{SOI}}(n)|^2 \right\rangle, \quad M_{\text{feed}} \times M_{\text{feed}} \text{ total interference ACM}$$

$$\rightarrow \mathbf{R}_{\varepsilon_{\text{ADC}} \varepsilon_{\text{ADC}}} + \mathbf{H}_{\text{SNOI}} \mathbf{R}_{s_{\text{SNOI}} s_{\text{SNOI}}} \mathbf{H}_{\text{SNOI}}^H$$

$$\mathbf{R}_{s_{\text{SNOI}} s_{\text{SNOI}}} \triangleq \left\langle |\mathbf{s}_{\text{SNOI}}(n)|^2 \right\rangle, \quad M_{\text{feed}} \times M_{\text{feed}} \text{ SNOI ACM}$$

$$\rightarrow \text{diag} \left\{ R_{s_{\text{SNOI}} s_{\text{SNOI}}}(\ell) \right\},$$

$$R_{s_{\text{SNOI}} s_{\text{SNOI}}}(\ell) \triangleq \left\langle |s_{\text{SNOI}}(n; \ell)|^2 \right\rangle \quad (\text{uncorrelated SNOIs})$$



## Maximum Attainable SINR (Max-SINR)

$$\begin{aligned}\gamma_{\text{SOI}} &= \frac{\left| \mathbf{w}_{\text{SOI}}^H \mathbf{h}_{\text{SOI}} \right|^2 R_{s_{\text{SOI}} s_{\text{SOI}}}}{\mathbf{w}_{\text{SOI}}^H \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}} \mathbf{w}_{\text{SOI}}} \\ &= \frac{\left| \mathbf{w}_{\text{SOI}}^H \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}} \left( \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}}^{-1} \mathbf{h}_{\text{SOI}} \right) \right|^2}{\left( \mathbf{w}_{\text{SOI}}^H \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}} \mathbf{w}_{\text{SOI}} \right) \left( \left( \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}}^{-1} \mathbf{h}_{\text{SOI}} \right)^H \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}} \left( \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}}^{-1} \mathbf{h}_{\text{SOI}} \right) \right)} \left( \left( \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}}^{-1} \mathbf{h}_{\text{SOI}} \right)^H \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}} \left( \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}}^{-1} \mathbf{h}_{\text{SOI}} \right) \right) R_{s_{\text{SOI}} s_{\text{SOI}}} \\ &\leq \left( \left( \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}}^{-1} \mathbf{h}_{\text{SOI}} \right)^H \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}} \left( \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}}^{-1} \mathbf{h}_{\text{SOI}} \right) \right) R_{s_{\text{SOI}} s_{\text{SOI}}} \quad (\text{Cauchy-Schwarz inequality}) \\ &= \mathbf{h}_{\text{SOI}}^H \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}}^{-1} \mathbf{h}_{\text{SOI}} R_{s_{\text{SOI}} s_{\text{SOI}}} \\ &\triangleq \gamma_{\max} \quad (\text{max-SINR}) \\ \gamma_{\text{SOI}} &= \gamma_{\max} \text{ iff } \mathbf{w}_{\text{SOI}} \propto \mathbf{R}_{i_{\text{SOI}} i_{\text{SOI}}}^{-1} \mathbf{h}_{\text{SOI}} \quad (\text{max-SINR combiner weights})\end{aligned}$$



## Interpretation for Antenna Array

$$\begin{aligned}
 \gamma_{\max} &= \mathbf{a}_{\text{ANT}}^H(\theta_{\text{SOI}}) \left( \mathbf{R}_{\boldsymbol{\epsilon}_{\text{ADC}} \boldsymbol{\epsilon}_{\text{ADC}}} + \sum_{\ell=1}^{L_{\text{SNOI}}} \mathbf{a}_{\text{ANT}}(\theta_{\text{SNOI}}(\ell)) \mathbf{a}_{\text{ANT}}^H(\theta_{\text{SNOI}}(\ell)) |g_{\text{TR-SNOI}}(\ell)|^2 R_{s_{\text{SNOI}} s_{\text{SNOI}}}(\ell) \right) \mathbf{a}_{\text{ANT}}(\theta_{\text{SOI}}) \\
 &\quad \times |g_{\text{TR-SOI}}|^2 R_{s_{\text{SOI}} s_{\text{SOI}}} \\
 &= \mathbf{v}_{\text{ANT}}^H(\theta_{\text{SOI}}) \left( \mathbf{I}_{M_{\text{feed}}} + \sum_{\ell=1}^{L_{\text{SNOI}}} \mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}}(\ell)) \mathbf{v}_{\text{ANT}}^H(\theta_{\text{SNOI}}(\ell)) \gamma_{\text{Rx-SNOI}}(\ell) \right)^{-1} \mathbf{v}_{\text{ANT}}(\theta_{\text{SOI}}) \gamma_{\text{Rx-SOI}} \\
 \mathbf{v}_{\text{ANT}}(\theta) &= \mathbf{C}_{\boldsymbol{\epsilon}_{\text{ADC}}}^H \mathbf{a}_{\text{ANT}}(\theta) \Big/ \sqrt{\frac{G_{\text{iso}}(\theta)}{R_{\boldsymbol{\epsilon}_{\text{iso}} \boldsymbol{\epsilon}_{\text{iso}}}}} \\
 \gamma_{\text{Rx-}(\bullet)} &\triangleq \frac{G_{\text{iso}}(\theta_{\text{Rx-}(\bullet)})}{R_{\boldsymbol{\epsilon}_{\text{iso}} \boldsymbol{\epsilon}_{\text{iso}}}} |g_{\text{TR-}(\bullet)}|^2 R_{s_{(\bullet)} s_{(\bullet)}}, \\
 \left\{ \begin{array}{l} \mathbf{C}_{\boldsymbol{\epsilon}_{\text{ADC}}} \triangleq \mathbf{R}_{\boldsymbol{\epsilon}_{\text{ADC}}}^{-1} \\ \mathbf{R}_{\boldsymbol{\epsilon}_{\text{ADC}}} \triangleq \text{chol}\{\mathbf{R}_{\boldsymbol{\epsilon}_{\text{ADC}} \boldsymbol{\epsilon}_{\text{ADC}}}\} \\ G_{\text{iso}}(\theta) = \text{isotropic reference beamformer gain} \\ (G_{\text{iso}}(\theta) \approx \text{constant over } \theta) \\ R_{\boldsymbol{\epsilon}_{\text{iso}} \boldsymbol{\epsilon}_{\text{iso}}} = \text{noise generated at output of isotropic beamformer} \end{array} \right. \\
 \text{Receive signal - to - noise ratio (SNR) using an isotropic reference beamformer}
 \end{aligned}$$



## Solution for Single SNOI

$$\begin{aligned}\gamma_{\max} &= \mathbf{v}_{\text{ANT}}^H(\theta_{\text{SOI}}) \left( \mathbf{I}_{M_{\text{feed}}} + \mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}}) \mathbf{v}_{\text{ANT}}^H(\theta_{\text{SNOI}}) \gamma_{\text{Rx-SNOI}} \right)^{-1} \mathbf{v}_{\text{ANT}}(\theta_{\text{SOI}}) \gamma_{\text{Rx-SOI}} \\ &= \mathbf{v}_{\text{ANT}}^H(\theta_{\text{SOI}}) \left( \mathbf{I}_{M_{\text{feed}}} - \frac{\mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}}) \mathbf{v}_{\text{ANT}}^H(\theta_{\text{SNOI}}) \gamma_{\text{Rx-SNOI}}}{1 + \|\mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}})\|^2 \gamma_{\text{Rx-SNOI}}} \right) \mathbf{v}_{\text{ANT}}(\theta_{\text{SOI}}) \gamma_{\text{Rx-SOI}} \quad (\text{Woodbury's identity}) \\ &= \left( \|\mathbf{v}_{\text{ANT}}(\theta_{\text{SOI}})\|^2 - \frac{|\mathbf{v}_{\text{ANT}}^H(\theta_{\text{SOI}}) \mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}})|^2 \gamma_{\text{Rx-SNOI}}}{1 + \|\mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}})\|^2 \gamma_{\text{Rx-SNOI}}} \right) \gamma_{\text{Rx-SOI}} \\ &= \left( 1 - \left( \frac{|\mathbf{v}_{\text{ANT}}^H(\theta_{\text{SOI}}) \mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}})|^2}{\|\mathbf{v}_{\text{ANT}}(\theta_{\text{SOI}})\|^2 \|\mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}})\|^2} \right) \left( \frac{\|\mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}})\|^2 \gamma_{\text{Rx-SNOI}}}{1 + \|\mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}})\|^2 \gamma_{\text{Rx-SNOI}}} \right) \right) \times \|\mathbf{v}_{\text{ANT}}(\theta_{\text{SOI}})\|^2 \gamma_{\text{Rx-SOI}}\end{aligned}$$

Reduction in SINR due to nulling of SNOI      Max-SNR beam-steerer



## Solution for Single SNOI

$$\begin{aligned}\gamma_{\max} &= \left(1 - |\rho(\theta_{\text{SOI}}, \theta_{\text{SNOI}})|^2 \left( \frac{G_{\max}(\theta_{\text{SNOI}}) \gamma_{\text{Rx-SNOI}}}{1 + G_{\max}(\theta_{\text{SNOI}}) \gamma_{\text{Rx-SNOI}}} \right)\right) \times G_{\max}(\theta_{\text{SOI}}) \gamma_{\text{Rx-SOI}} \\ &\rightarrow \left(1 - |\rho(\theta_{\text{SOI}}, \theta_{\text{SNOI}})|^2\right) \times G_{\max}(\theta_{\text{SOI}}) \gamma_{\text{Rx-SOI}}, \quad \gamma_{\text{Rx-SNOI}} \rightarrow \infty\end{aligned}$$

$$\rho(\theta_{\text{SOI}}, \theta_{\text{SNOI}}) \triangleq \left( \frac{\mathbf{v}_{\text{ANT}}(\theta_{\text{SOI}})}{\|\mathbf{v}_{\text{ANT}}(\theta_{\text{SOI}})\|} \right)^H \left( \frac{\mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}})}{\|\mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}})\|} \right),$$

Generalized cosine between  
 $\mathbf{v}_{\text{ANT}}(\theta_{\text{SOI}})$  and  $\mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}})$

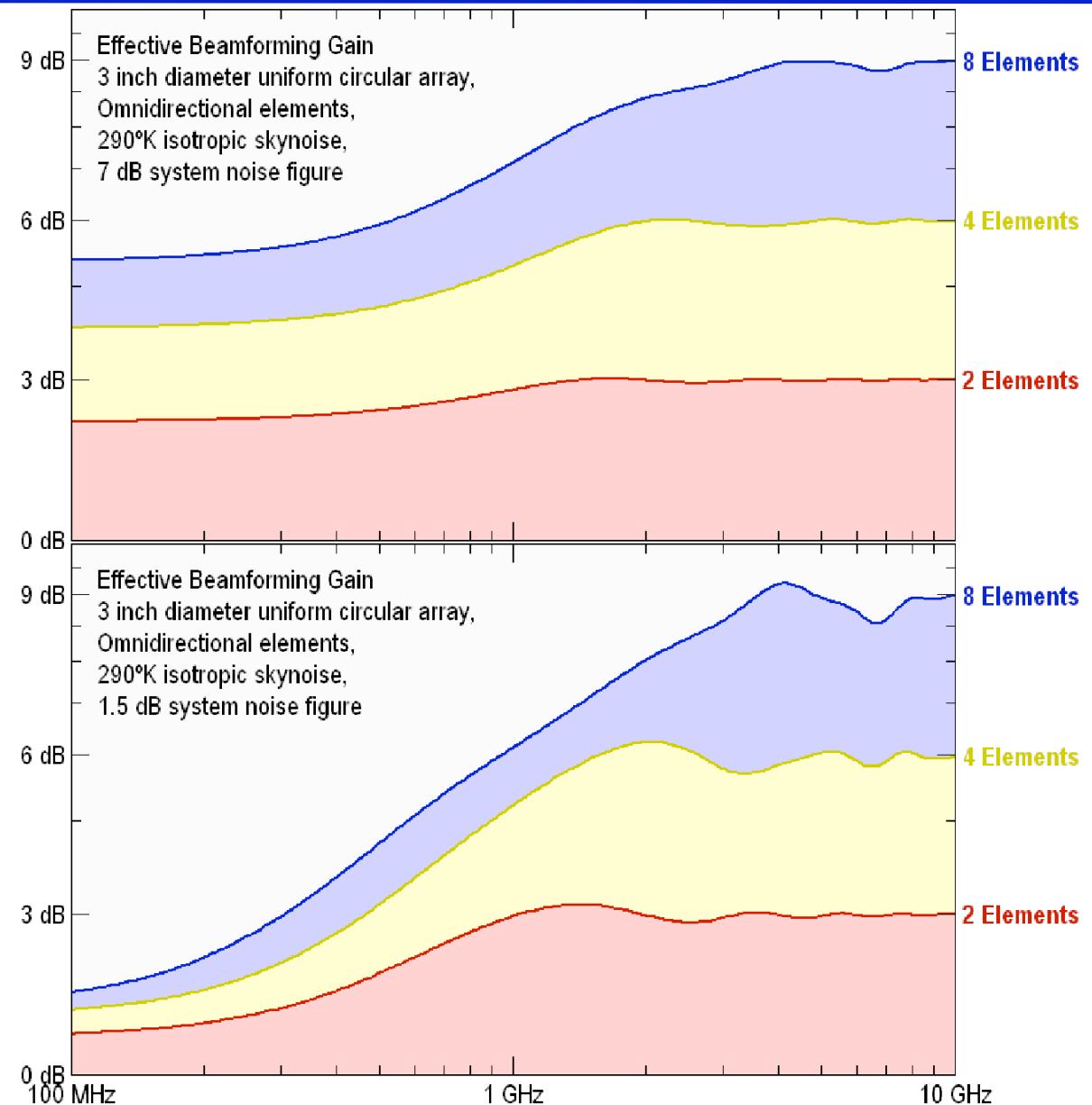
$$\begin{aligned}G_{\max}(\theta) &\triangleq \|\mathbf{v}_{\text{ANT}}(\theta)\|^2 \\ &= \left( \mathbf{a}_{\text{ANT}}^H(\theta) \mathbf{R}_{\epsilon_{\text{ADC}} \epsilon_{\text{ADC}}}^{-1} \mathbf{a}_{\text{ANT}}(\theta) \right) \times \frac{R_{\epsilon_{\text{iso}} \epsilon_{\text{iso}}}}{G_{\text{iso}}(\theta)},\end{aligned}$$

Maximum attainable SNR (max-SNR)  
beam-steering gain over isotropic at DOA  $\theta$

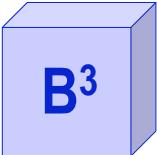
$$\begin{aligned}\langle G_{\max}(\theta) \rangle &\rightarrow \text{Tr} \left\{ \left( \left( 1 - \frac{1}{F_{\text{sys}}} \right) \mathbf{I}_{M_{\text{feed}}} + \frac{1}{F_{\text{sys}}} \mathbf{R}_{\mathbf{a}_{\text{ANT}} \mathbf{a}_{\text{ANT}}} \right)^{-1} \mathbf{R}_{\mathbf{a}_{\text{ANT}} \mathbf{a}_{\text{ANT}}} \right\}, \quad \mathbf{R}_{\mathbf{a}_{\text{ANT}} \mathbf{a}_{\text{ANT}}} = \langle \mathbf{a}_{\text{ANT}}(\theta) \mathbf{a}_{\text{ANT}}^H(\theta) \rangle \\ &\rightarrow \begin{cases} M_{\text{feed}}, & \text{Moderate-to-large electrical array aperture} \\ \frac{M_{\text{feed}} F_{\text{sys}}}{M_{\text{feed}} + F_{\text{sys}} - 1}, & \text{Small electrical array aperture } (d_{\text{ANT}} \ll \lambda_{\text{Rx}}) \end{cases}\end{aligned}$$

# Average Max-SNR Beamforming Gain vs. Frequency, 3 Inch UCA, Noise-Limited Environment

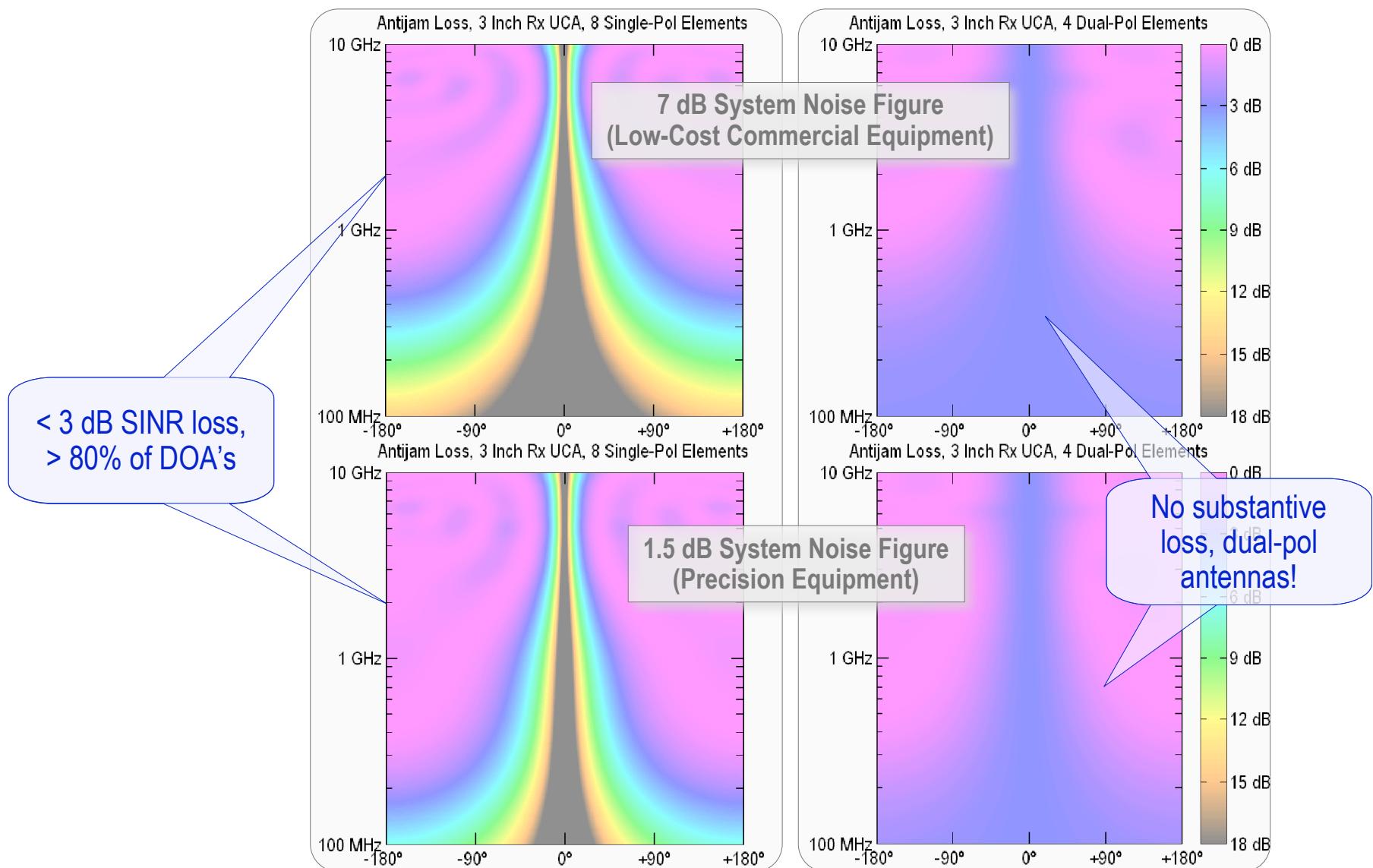
7 dB System Noise Figure  
(Low-Cost Commercial Equipment)



1.5 dB System Noise Figure  
(Precision Equipment)

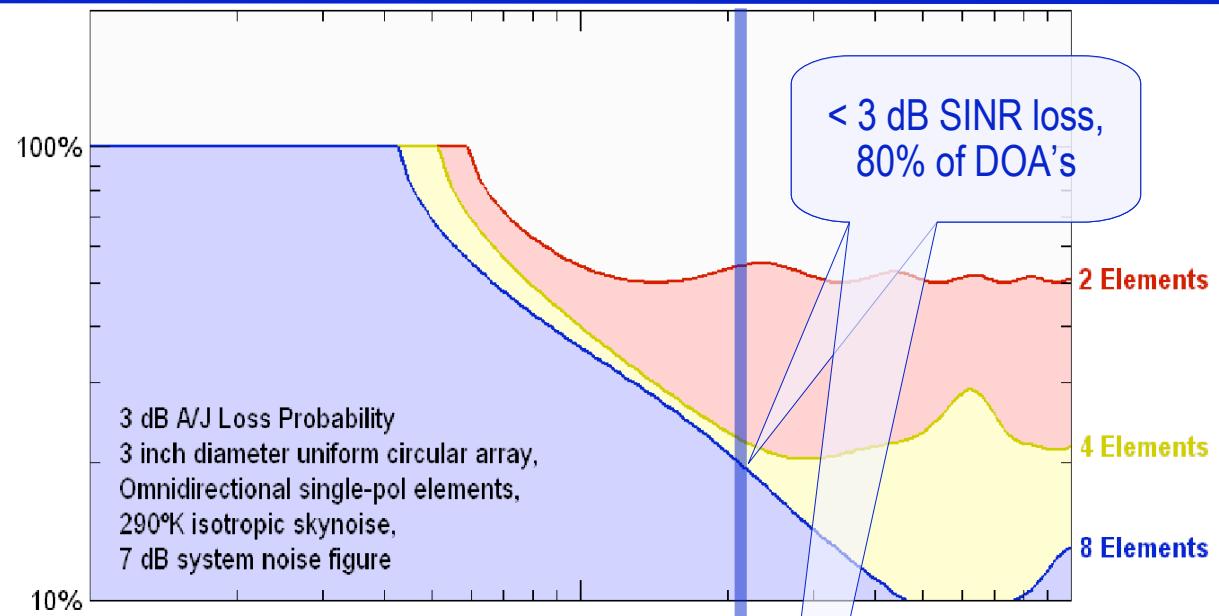


# Reduction from Max-SNR (Antijam Loss), Infinite SNOI Power 3 Inch UCA, 8 Adaptive Feeds, Noise-Limited Environment

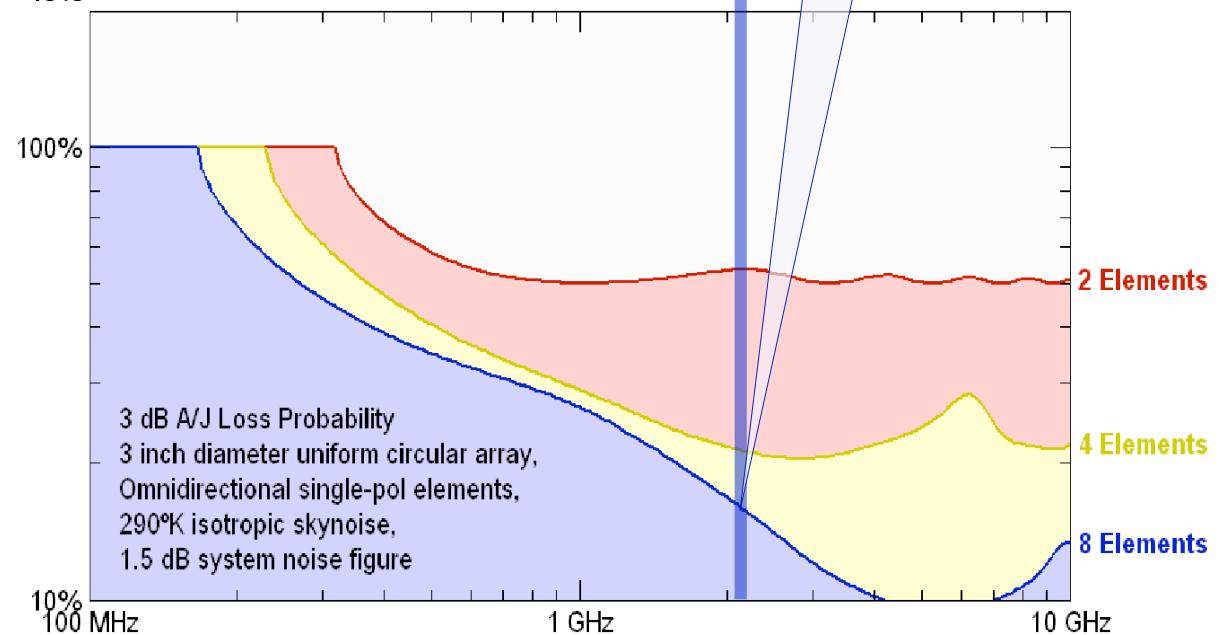


# Probability > 3 dB Antijam Loss, 3 Inch UCA, Single-Pol Antennas, Noise-Limited Environment

7 dB System Noise Figure  
(Low-Cost Commercial Equipment)



1.5 dB System Noise Figure  
(Precision Equipment)





## Extension to Multiple SNOI's

$$\begin{aligned}\gamma_{\max} &= \mathbf{v}_{\text{ANT}}^H(\theta_{\text{SOI}}) \left( \mathbf{I}_{M_{\text{feed}}} + \mathbf{v}_{\text{ANT}}(\theta_{\text{SNOI}}) \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \mathbf{v}_{\text{ANT}}^H(\theta_{\text{SNOI}}) \right)^{-1} \mathbf{v}_{\text{ANT}}(\theta_{\text{SOI}}) \gamma_{\text{Rx-SOI}} \\ &= \mathbf{v}_{\text{SOI}}^H \left( \mathbf{I}_{M_{\text{feed}}} - \mathbf{v}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \left( \mathbf{I}_{L_{\text{SNOI}}} + \mathbf{v}_{\text{SNOI}}^H \mathbf{v}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \right)^{-1} \mathbf{v}_{\text{SNOI}}^H \right) \mathbf{v}_{\text{SOI}} \gamma_{\text{Rx-SOI}} \quad (\text{Matrix Inversion Lemma}) \\ &= \left( 1 - \left( \frac{\mathbf{v}_{\text{SOI}}}{\|\mathbf{v}_{\text{SOI}}\|} \right)^H \left( \mathbf{v}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \left( \mathbf{I}_{L_{\text{SNOI}}} + \mathbf{v}_{\text{SNOI}}^H \mathbf{v}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \right)^{-1} \mathbf{v}_{\text{SNOI}}^H \right) \left( \frac{\mathbf{v}_{\text{SOI}}}{\|\mathbf{v}_{\text{SOI}}\|} \right) \right) \times \|\mathbf{v}_{\text{SOI}}\|^2 \gamma_{\text{Rx-SOI}}\end{aligned}$$

Reduction in SINR due to nulling of all SNOI's

Max-SNR beam-steerer

## Extension to Multiple SNOI's

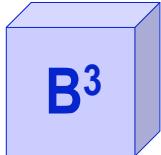
$$\begin{aligned}
 \langle \gamma_{\max}(\theta_{\text{SOI}}) \rangle &= \text{Tr} \left\{ \left( \mathbf{I}_{M_{\text{feed}}} - \mathbf{v}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \left( \mathbf{I}_{L_{\text{SNOI}}} + \mathbf{v}_{\text{SNOI}}^H \mathbf{v}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \right)^{-1} \mathbf{v}_{\text{SNOI}}^H \right) \langle \mathbf{v}_{\text{ANT}}(\theta_{\text{SOI}}) \mathbf{v}_{\text{ANT}}^H(\theta_{\text{SOI}}) \rangle \right\} \gamma_{\text{Rx-SOI}} \\
 &\rightarrow \text{Tr} \left\{ \mathbf{I}_{M_{\text{feed}}} - \mathbf{v}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \left( \mathbf{I}_{L_{\text{SNOI}}} + \mathbf{v}_{\text{SNOI}}^H \mathbf{v}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \right)^{-1} \mathbf{v}_{\text{SNOI}}^H \right\} \gamma_{\text{Rx-SOI}}, \quad (F_{\text{sys}} \rightarrow 1) \\
 &= \left( M_{\text{feed}} - \text{Tr} \left\{ \left( \mathbf{I}_{L_{\text{SNOI}}} + \mathbf{v}_{\text{SNOI}}^H \mathbf{v}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \right)^{-1} \mathbf{v}_{\text{SNOI}}^H \mathbf{v}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \right\} \right) \gamma_{\text{Rx-SOI}} \\
 &= \left( M_{\text{feed}} - \text{Tr} \left\{ \mathbf{I}_{L_{\text{SNOI}}} - \left( \mathbf{I}_{L_{\text{SNOI}}} + \mathbf{v}_{\text{SNOI}}^H \mathbf{v}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \right)^{-1} \right\} \right) \gamma_{\text{Rx-SOI}} \\
 &= \left( M_{\text{feed}} - L_{\text{SNOI}} + \text{Tr} \left\{ \left( \mathbf{I}_{L_{\text{SNOI}}} + \mathbf{v}_{\text{SNOI}}^H \mathbf{v}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \right)^{-1} \right\} \right) \gamma_{\text{Rx-SOI}} \\
 &\geq (M_{\text{feed}} - L_{\text{SNOI}}) \gamma_{\text{Rx-SOI}}
 \end{aligned}$$

- 1 combiner degree of freedom (DoF) used to excise each SNOI
- Remaining DoF's used to increase SOI SNR (suppress noise)

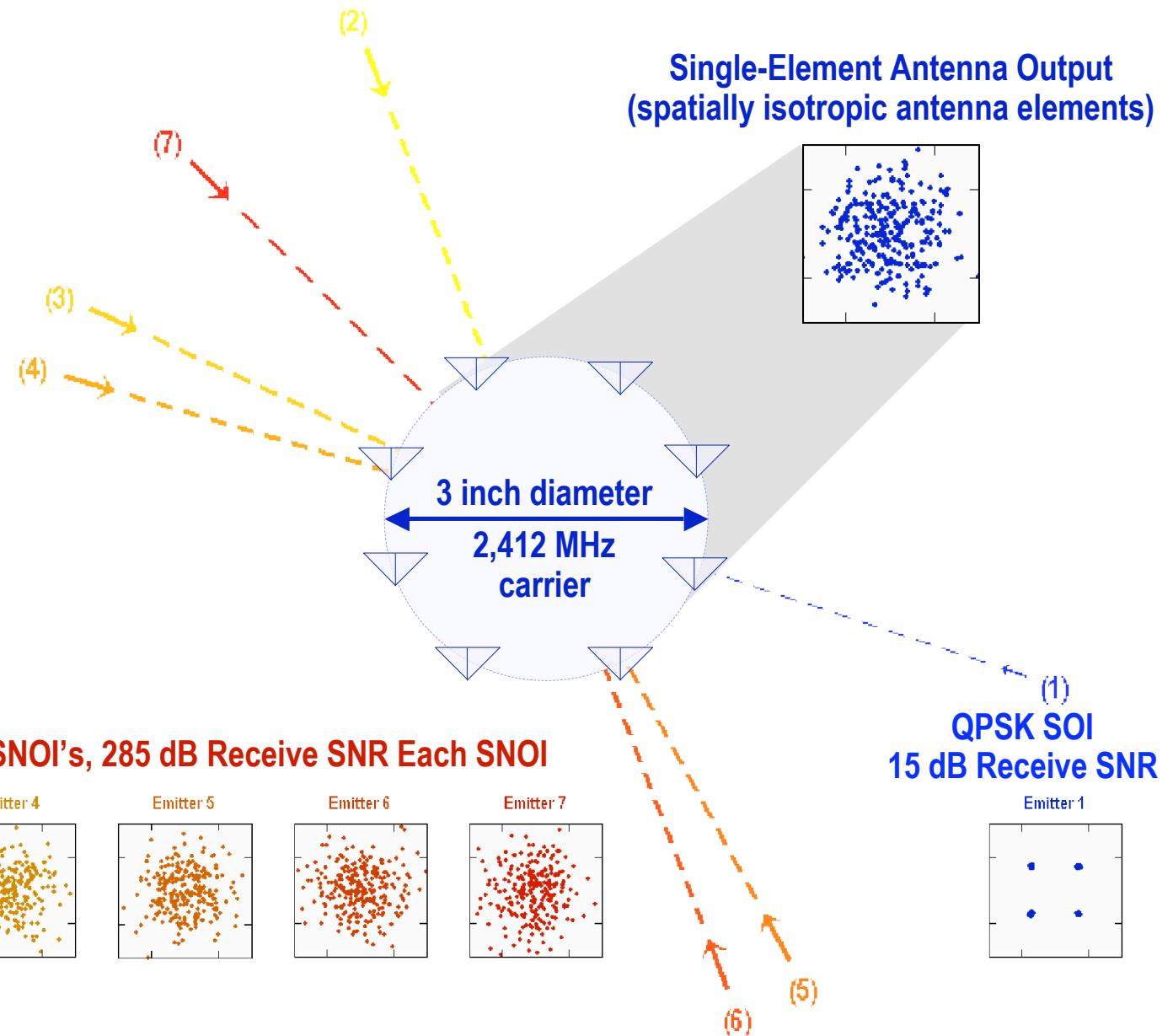
## Extension to Multiple SNOI's

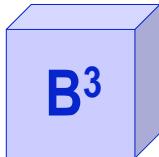
$$\begin{aligned}
 \langle \gamma_{\max}(\theta_{\text{SOI}}) \rangle &\rightarrow \left( M_{\text{feed}} - L_{\text{SNOI}} + \text{Tr} \left\{ \left( \mathbf{I}_{L_{\text{SNOI}}} + \mathbf{V}_{\text{SNOI}}^H \mathbf{V}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \right)^{-1} \right\} \right) \gamma_{\text{Rx-SOI}} \\
 &\geq \left( M_{\text{feed}} - L_{\text{SNOI}} + \frac{L_{\text{SNOI}}^2}{\text{Tr} \left\{ \mathbf{I}_{L_{\text{SNOI}}} + \mathbf{V}_{\text{SNOI}}^H \mathbf{V}_{\text{SNOI}} \boldsymbol{\Lambda}_{\text{Rx-SNOI}} \right\}} \right) \gamma_{\text{Rx-SOI}}, \quad \text{Tr}(\mathbf{R}^{-1}) \geq \frac{L^2}{\text{Tr}(\mathbf{R})}, \quad \mathbf{R} > 0, L \times L \\
 &= \left( M_{\text{feed}} - L_{\text{SNOI}} + \frac{L_{\text{SNOI}}}{1 + \frac{1}{L_{\text{SNOI}}} \sum_{\ell=1}^{L_{\text{SNOI}}} G_{\max}(\theta_{\text{SNOI}}(\ell)) \gamma_{\text{Rx-SNOI}}(\ell)} \right) \gamma_{\text{Rx-SOI}} \\
 &= \left( M_{\text{feed}} - L_{\text{SNOI}} \frac{\langle \gamma_{\max-\text{SNR}}(\ell) \rangle}{1 + \langle \gamma_{\max-\text{SNR}}(\ell) \rangle} \right) \gamma_{\text{Rx-SOI}}
 \end{aligned}$$

Soft excision of SNOI's with finite receive SNR

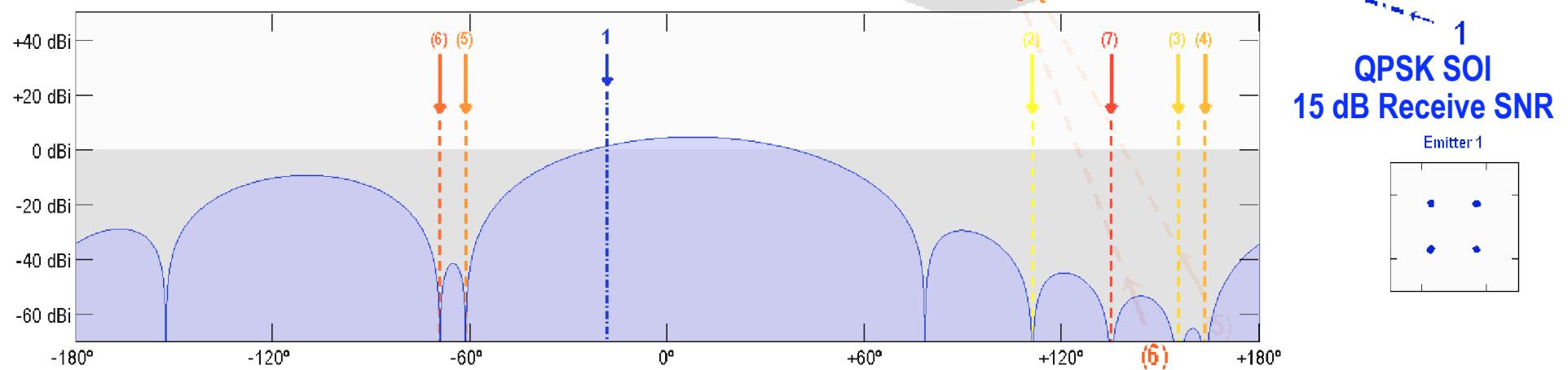
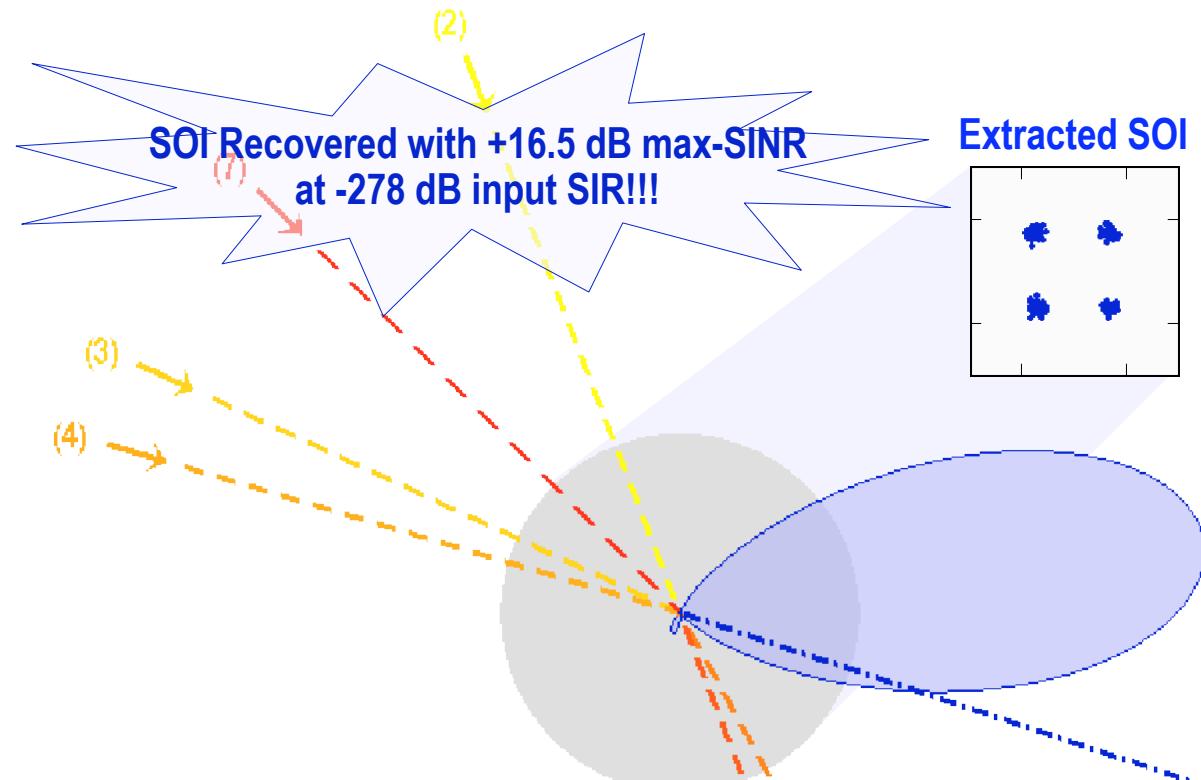


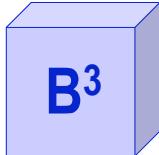
# Extreme Excision Example: 15 dB SOI Receive SNR, 270 dB SOI/SNOI Interference Margin



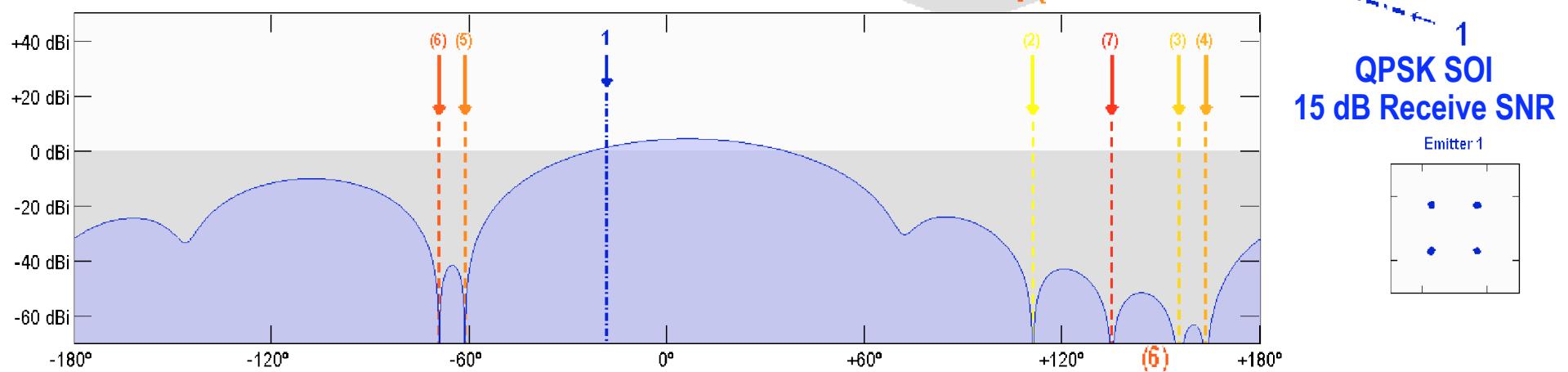
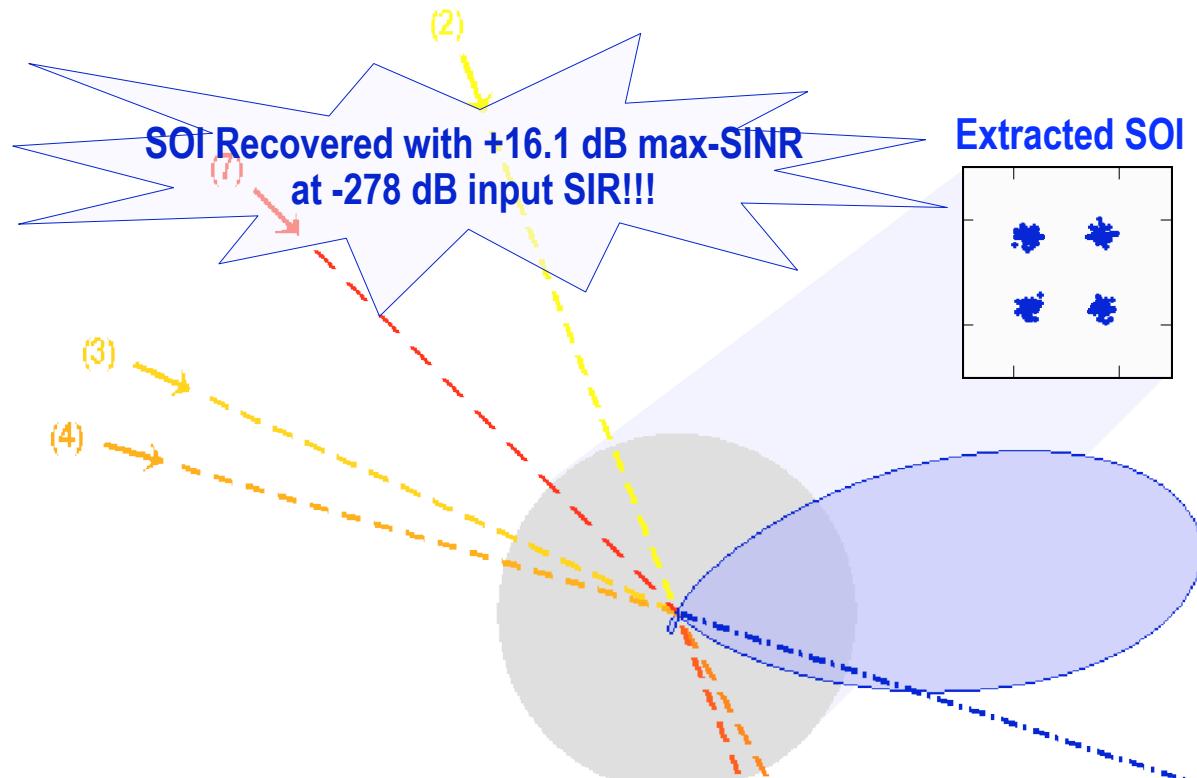


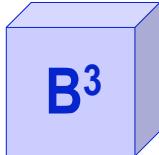
## Ideal Maximum-SINR Solution



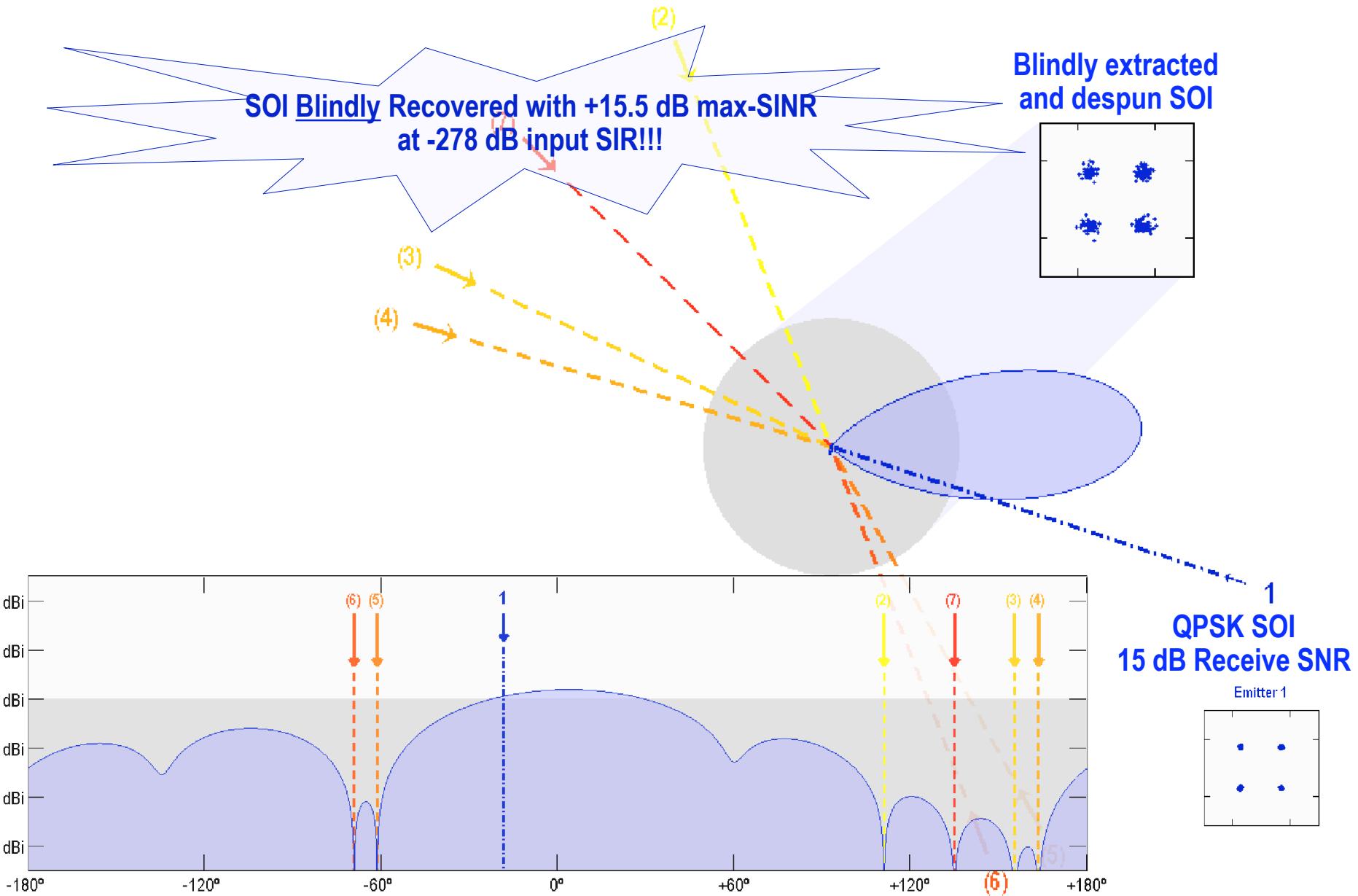


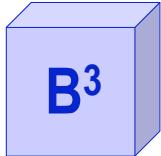
# Nonblind Adaptation Algorithm Performance, 256-Symbol TBP (FFT Least-Squares)



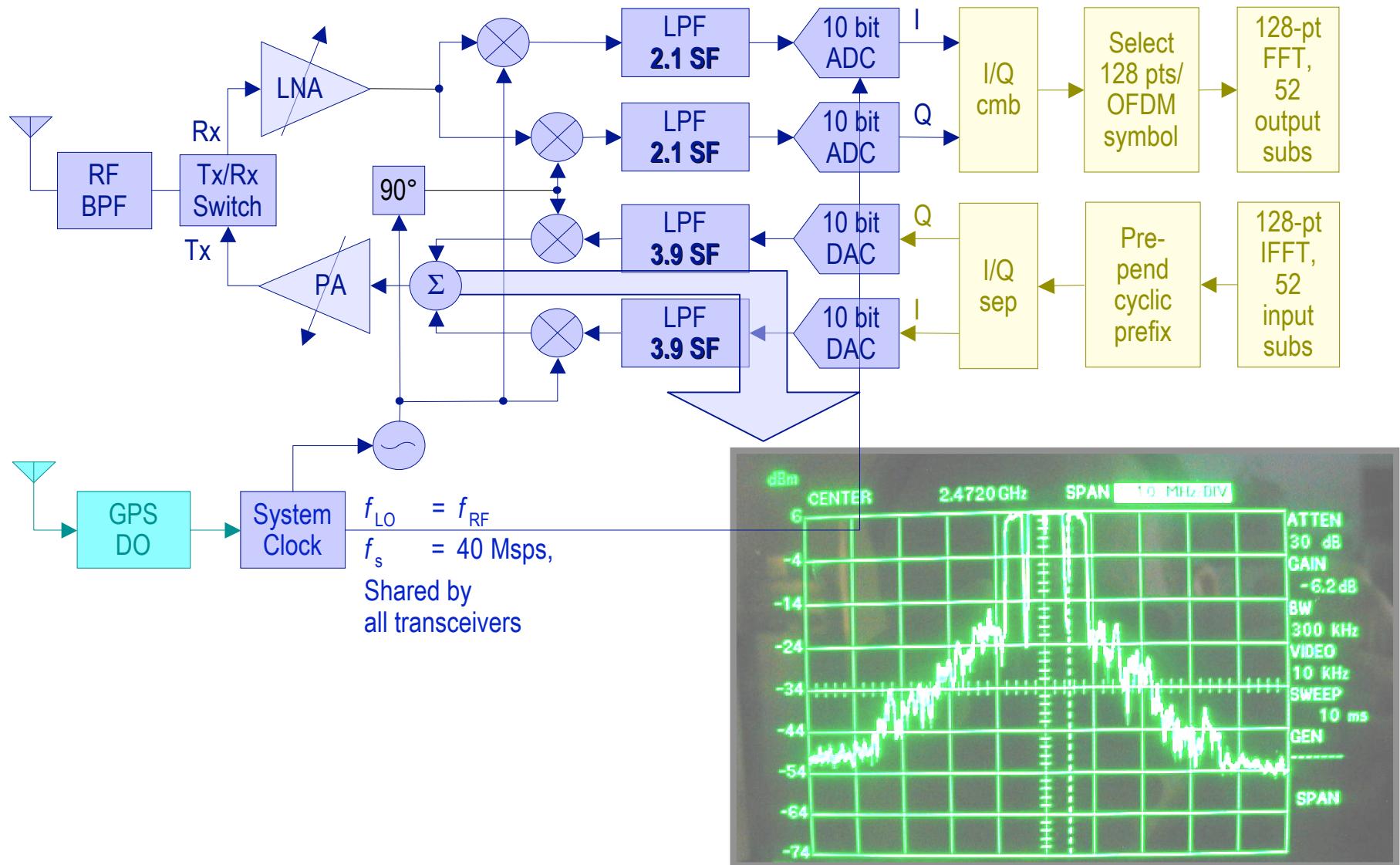


# Blind Adaptation Algorithm Performance, 256 Symbol TBP (Multitarget Least-Squares CMA, 1989)

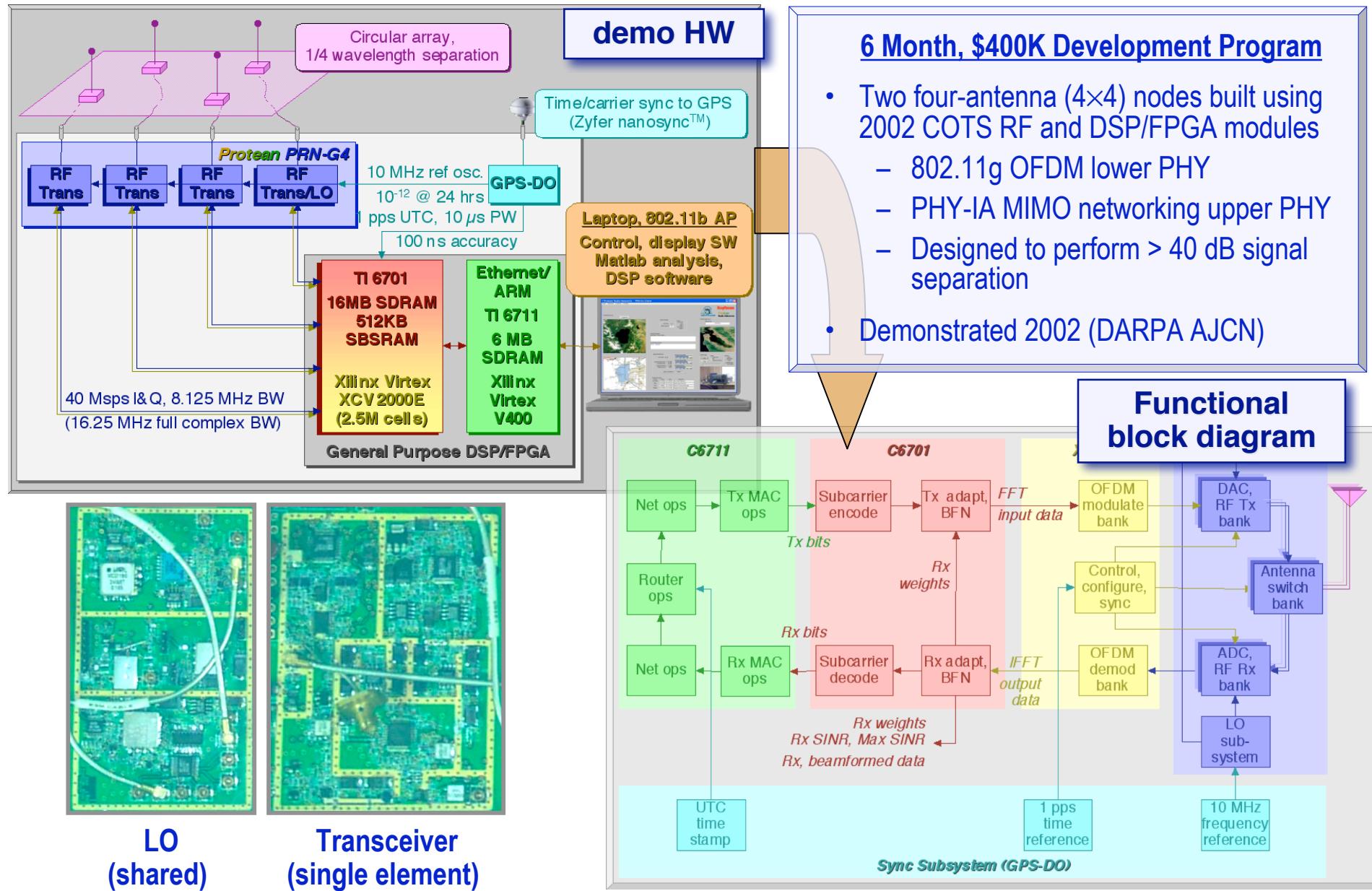




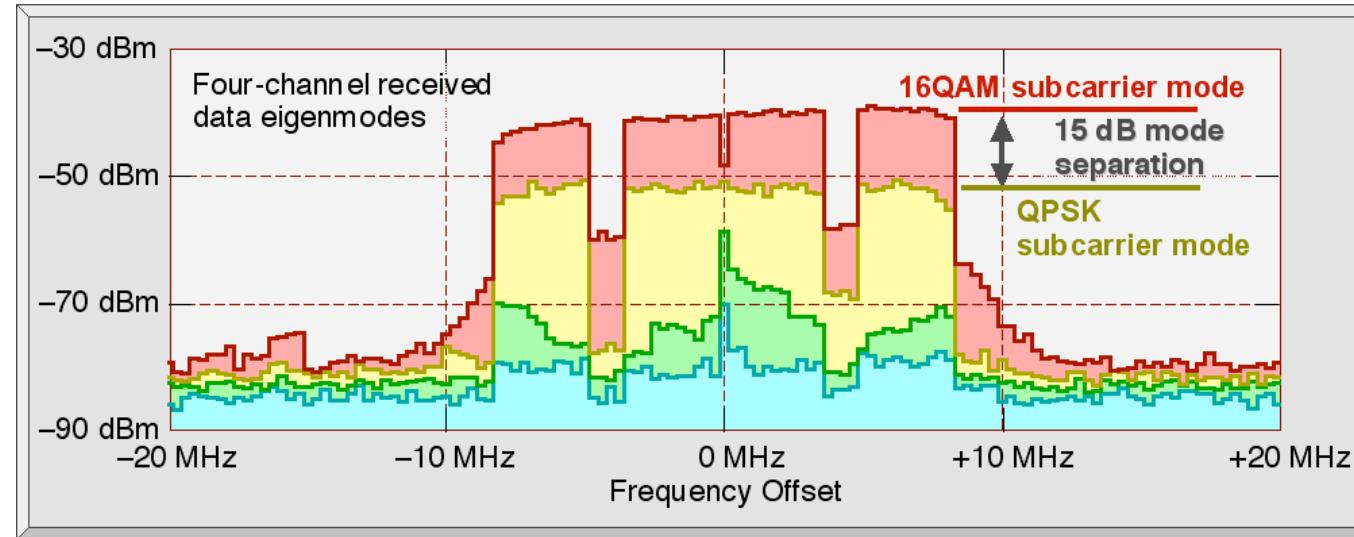
# Real Excision Example, 802.11g Compatible OFDM Transceiver (Protean Radio Networks, 2002)



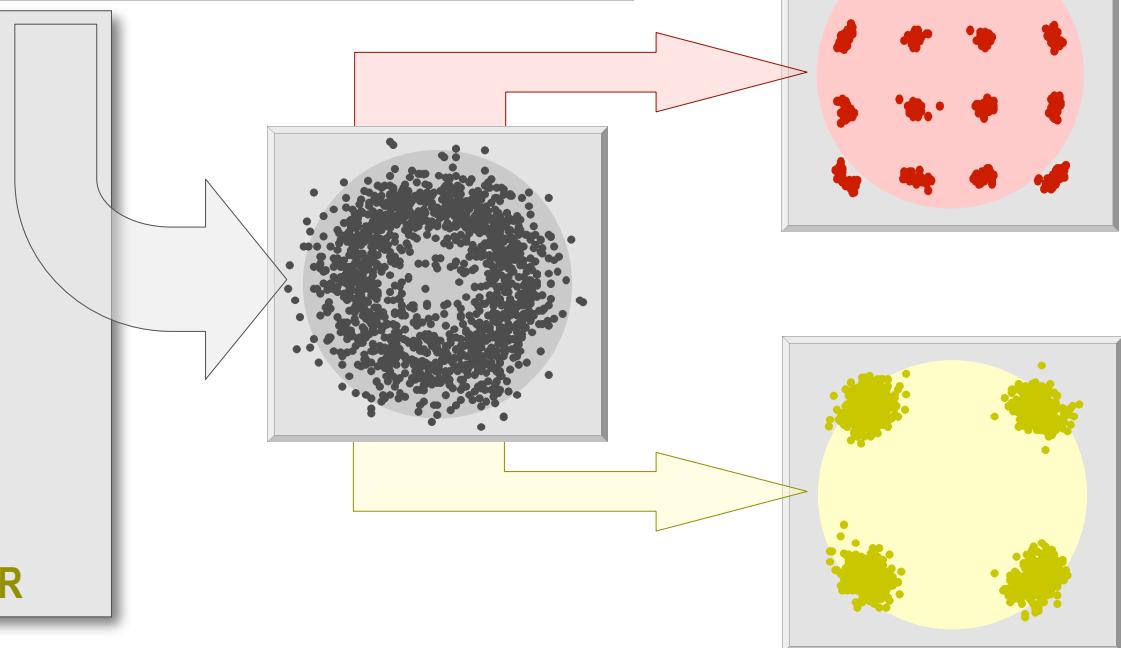
# Demonstration System



## Adaptive Data Separation, $\pm 6$ dB SIR

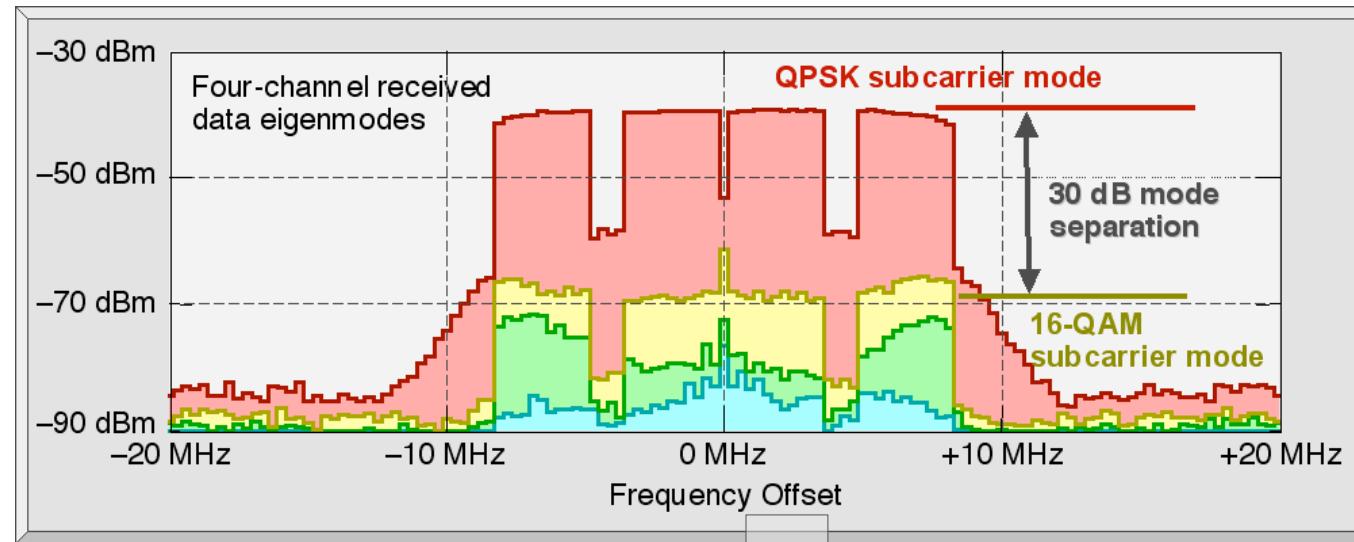


- Two 802.11g OFDM signals received with 6 dB separation
  - 4 bps/Hz signal (16QAM subcarriers) received at +6 dB SINR
  - 2 bps/Hz signal (QPSK subcarriers) received at -6 dB SINR
- Signals separated without bit errors
  - ✓ 4 bps/Hz signal (16QAM subcarriers) corrected to +24 dB beamformed SINR
  - ✓ 2 bps/Hz signal (QPSK subcarriers) corrected to +17 dB beamformed SINR



B<sup>3</sup>

## Adaptive Data Separation, $\pm 30$ dB SIR



- Two 802.11g OFDM signals received with 30 dB separation
  - 2 bps/Hz signal (QPSK subcarriers) received at **+30 dB SINR**
  - 4 bps/Hz signal (16QAM subcarriers) received at **-30 dB SINR**
- Signals separated with low bit error
  - ✓ QPSK subcarriers corrected to **+17 dB beamformed SINR** — no errors
  - ✓ 16QAM subcarriers corrected to **+17 dB beamformed SINR** — **0.4% raw BER** (**no errors after decoding**)

